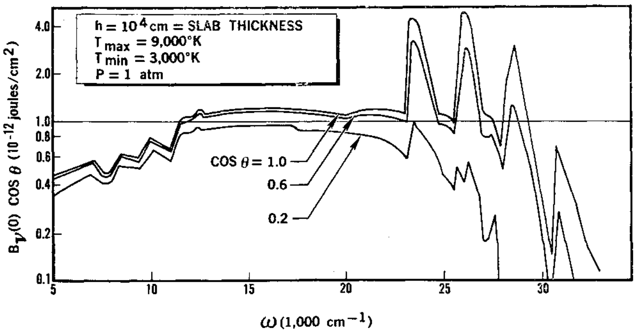

Fig. 1 The spectral radiancy incident on a cold wall from a slab of air with a parabolic temperature profile.

temperature ratio $\theta_0(=T_{\text{cold wall}}/T_{\max})$, and the optical thickness τ_0. The effect of variation of these parameters upon the solution obtained by the use of Eq. (1) is demonstrated in Fig. 2, where the parameter θ_0 is varied. For a given τ_0, Bu_P was the same for all N and θ_0, and Bu_R was equal to 1.4 in all cases. The agreement between the approximate and the exact temperature profiles was good over a wide range of τ_0, N, and θ_0.

In Table 1, the approximate and exact heat fluxes due to conduction and radiation are compared. The agreement is generally about 10%. Poor values of the heat fluxes are found for large θ_0 because the very nature of the temperature profile changes, requiring recomputation of Bu_P (not done in this work). Computing machine time for solution of Eq. (1) for the preceding system is about 10^{-2} times that for the exact solution for small $\tau_0(<1)$, and about 10^{-4} times that for the exact solution for moderate $\tau_0(1 < \tau_0 < 10)$. For the case of a nongray gas, the time savings would be considerably more.

The preceding analysis suggests the following:

1) Empirical equations such as Eqs. (1) and (2) yield heat-transfer results good to within 10%.

2) The critical Bouguer numbers, Bu_P and Bu_R, may be determined separately and accurately.

3) The Bouguer number associated with optically thick systems is nearly independent of the temperature profile and heat transfer parameters.

4) If spectral values of the emitted radiation are desired, it is possible to use Eq. (1) and the mean absorption coefficients to find the temperature profile and then to use exact theory to find the desired data with relative ease.

For cases of radiating or reflecting boundaries, Eqs. (1) and (2) must be modified. We are presently defining the changes appropriate for consideration of stagnation-point heat transfer during the atmospheric entry of very high velocity space vehicles.

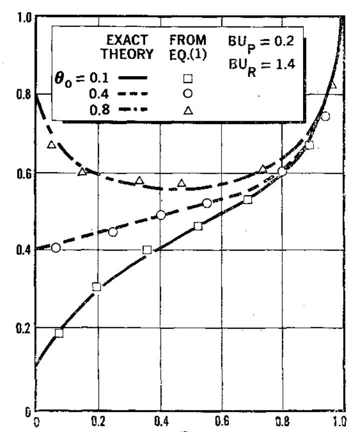

Fig. 2 A comparison of exact and approximate dimensionless temperature profiles for τ_0 = 1.0 and N = 0.01.

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Cone Pressure Distribution at Large and Small Angles of Attack

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Nomenclature

C_P = pressure coefficient
K = hypersonic similarity parameter, $M_1 \sin\delta_e$
M = Mach number
α = angle of attack
β = $(M_1^2 - 1)^{1/2}$
δ = half cone angle (semivertex angle)
Φ = circumferential angle (= 0 at bottom)

Subscripts

1 = freestream
c = cone surface
e = equivalent cone
s = flow separation

In 1961, Jacobs[1] developed an approximate method for calculating the pressure on arbitrarily-shaped conical bodies at zero incidence as well as at angle of attack. The method is limited to angles of attack less than the half cone angle when applied to right circular cones. The basic equation that Jacobs developed took the following form:

$$C_p = C_{p_e} - (C_{p_e} - C_{p_e}^*)f(M) \quad (1)$$

where C_{p_e} is the pressure coefficient on the equivalent cone (a right circular cone at zero incidence having the same freestream Mach number and a half cone angle equal to the angle between the freestream Mach number vector and the ray line on the cone in question) and $C_{p_e}^*$ was, according to Jacobs' method, the average value of C_{p_e} over the peripheral of the cone, and $f(M)$ was assumed to be equal to $1/M_c$.

$\alpha < \delta$: For the case of the angle of attack α less than the half cone angle δ, Jacobs' method gives excellent results. However, for computational ease, we have been able to modify his method slightly and still obtain good correlation with experimental data. The method consists of using the Linnell-

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Bailey equation[2] for pressure coefficient on cones for the C_{p_e} term in Eq. (1):

$$C_{p_e} = 4\sin^2\delta_e(2.5 + 8\beta\sin\delta_e)(1 + 16\beta\sin\delta_e)^{-1} \quad (2)$$

β is equal to $(M^2 - 1)^{1/2}$ and δ_e is the equivalent half cone angle at a given ray line (it is the angle between the ray line and the freestream Mach number vector). It is possible to show that

$$\delta_e = \sin^{-1}(\sin\delta\cos\alpha + \cos\delta\sin\alpha\cos\Phi) \quad (3)$$

Φ is the ray angle of the cone, with $\Phi = 0$ at the bottom ray line and 180° at the top ray line.

In our method $C_{p_e}{}^*$ is determined by evaluating C_{p_e} [Eq. (2)] at $\Phi = 90°$. We have also found that $f(M)$ can be expressed as

$$f(M) = (M_c{}^{-3/2})_{\Phi=90°} \quad (4)$$

where $(M_c)_{\Phi=90°}$ is the local equivalent cone Mach number (M_c) at $\Phi = 90°$. This can be easily and quickly calculated by Blick's approximate equation[3] (for δ_e evaluated at $\Phi = 90°$). For $K < 1$,

$$M_c/M_1 = \cos\delta_e(1 - \sin\delta_e/M_1)^{1/2} \times \{[1 + \exp(-1 - 1.52K)][1 + (K/2)^2]\}^{-1/2} \quad (5a)$$

or if $K \geq 1$,

$$M_c/M_1 = \cos\delta_s(1 - \sin\delta_e/M_1)^{1/2}(1 + 0.35K^{3/2})^{-1/2} \quad (5b)$$

where K is the hypersonic similarity parameter $M_1\sin\delta_e$. The present method for $\alpha < \delta$ consists of using Eqs. (2–5) in Eq. (1). Figure 1 shows an application of this method compared with experimental data for a right circular cone.

$\alpha > \delta$: The method specified by Jacobs does not work in the region where the angle of attack was greater than the half cone angle. Several modifications of Jacobs' method were accordingly attempted in order to produce a method suitable for this region.

Using Eq. (1) as the basic equation, it was found that the value of $f(M)$ for $\alpha > \delta$ should be different for each ray line (Φ = const line). The $f(M)$ variable used is

$$f(M) = (M_c{}^{-3/2})_\Phi \quad (6)$$

where $(M_c)_\Phi$ is the local equivalent cone Mach number (M_c) evaluated at the Φ (and hence local δ_e) under consideration, and it can be calculated by Blick's equations [Eq. (5)].

The value of $C_p{}^*$ for $\alpha > \delta$ was obtained by setting the pressure coefficient at the $\Phi = 0°$ ray line [using Eq. (1)] to the pressure coefficient predicted by Newtonian impact theory:

$$2\sin^2(\alpha + \delta) = C_{p_e} - (C_{p_e} - C_{p_e}{}^*)M_{c\Phi=0}{}^{-3/2} \quad (7)$$

Using the Linnell-Bailey equation [Eq. (2)] for C_{p_e} (evaluated at $\Phi = 0$), the expression for $C_p{}^*$ is then

$$C_{p_e}{}^* = M_{c\Phi=0}{}^{3/2}[2\sin^2(\alpha + \delta)] + (1 - M_{c\Phi=0}{}^{3/2})(C_{p_e})_{\Phi=0} \quad (8)$$

Unlike Jacobs' original method, $C_{p_e}{}^*$ given by Eq. (8) does not represent the average value of C_{p_e} over the body. The present method for $\alpha > \delta$ consists of using Eqs. (2, 3, 6, 5, 8) in Eq. (1) for ray lines up to the point where $\delta_e = 0°$. From this point to the separation ray line (Φ_s) a curve is faired in from the value of C_p evaluated by the foregoing method at $\delta_e = 0°$ to the value of C_p evaluated at Φ_s by assuming a Prandtl-Meyer expansion from the freestream Mach number through the angle δ_{e_s} [δ_e evaluated at Φ_s by using Eq. (2)].

A relation that is felt to give a fairly good representation of the separation ray line (Φ_s) is

$$\Phi_s = C_1/\alpha + C_2 \quad (9)$$

C_1 and C_2 are constants that may be determined from the approximate boundary conditions,

$$\alpha = \delta \qquad \Phi_s = 180°$$
$$\alpha = 90° \qquad \Phi_s = 110°$$

The second boundary condition represents the separation for flow normal to a long cylinder.

A comparison of this new method with available experiments on cones at large angles of attack indicates very good agreement. An example of this agreement is illustrated in Fig. 2.

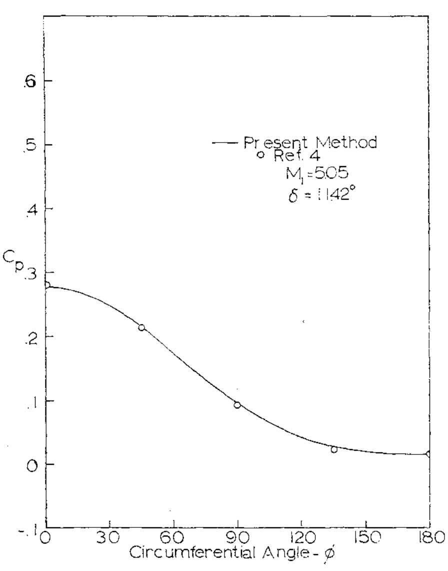

Fig. 1 Circular cone at $\alpha = 10°$.

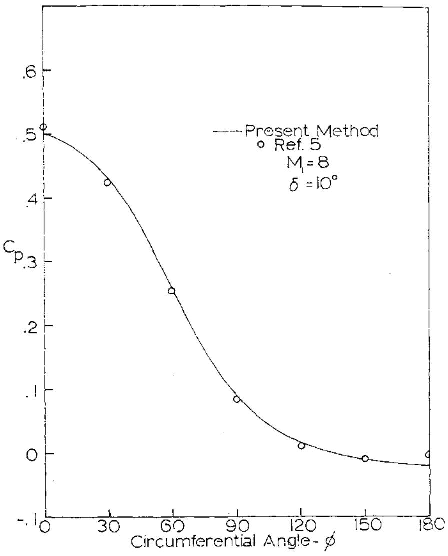

Fig. 2 Circular cone at $\alpha = 20°$.

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